Bjorken and Ellis-Jaffe Sumrules from R-evolution

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arxiv: 0908.3189 (hep-ph) + work in progress
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Outline

- Review of theory and expt. results
- Renormalon problem in $\overline{\text{MS}}$
- New MSR scheme and R-evolution
- Fits for Power corrections in MSR (and $\overline{\text{MS}}$)

Theory Review

Ellis-Jaffe Sum Rule (MS Scheme)

Ellis, Jaffe (1973)
Nachtmann (1973)

OPE for the Nachtmann moment of the nucleon structure function

$$M_1^{p/n}(Q) \quad = \quad \left[\pm \hat{C}_B(Q)\left(\frac{1}{12}g_A + \frac{1}{36}a_8\right) + \hat{C}_0(Q)\frac{\hat{a}_0}{9}\right] \boxed{\text{twist 2}}$$

sum of elastic and inelastic contribution

$$\boxed{\text{twist 4}} \quad -\frac{1}{Q^2}\left[\left(\pm\frac{2}{27}\hat{f}_3 + \frac{2}{81}\hat{f}_8\right)[\alpha_s(Q)]^{\gamma_0^{ns}/(2\beta_0)} + \frac{8}{81}\hat{f}_0\ [\alpha_s(Q)]^{\gamma_0^{s}/(2\beta_0)}\right]$$

$$+\frac{1}{Q^4}\left[\pm h_B + h_0\right] \boxed{\text{twist 6}}$$

- Renormalization scale independent Wilson coefficients and matrix elements $\quad \bar{C}(Q,\mu)\bar{\theta}(Q,\mu) = \hat{C}(Q)\hat{\theta}$
- At twist 2: matrix elements of (flavor) non-singlet axial current $\quad J_\mu^{5a} = \bar{\psi}\gamma_\mu\gamma^5 t^a\psi(x)$
- At twist 4: reduced matrix elements of $R_{2\sigma}^a = g\bar{\psi}\tilde{G}_{\sigma\rho}\gamma^\rho t^a\psi$
- twist 6: important to fit the data

for details, see for example: Campanario, Pineda (2005)

Theory Review

Bjorken Sum Rule ($\overline{\text{MS}}$ Scheme)

$$M_1^B \equiv M_1^p(Q) - M_1^n(Q) \quad = \quad \underbrace{\hat{C}_B(Q)\,\frac{g_A}{6}}_{\text{twist 2}} - \underbrace{\frac{4[\alpha_s(Q)]^{\gamma_0^{ns}/(2\beta_0)}}{27Q^2}\hat{f}_3}_{\text{twist 4}} + \underbrace{\frac{1}{Q^4}h_B}_{\text{twist 6}}$$

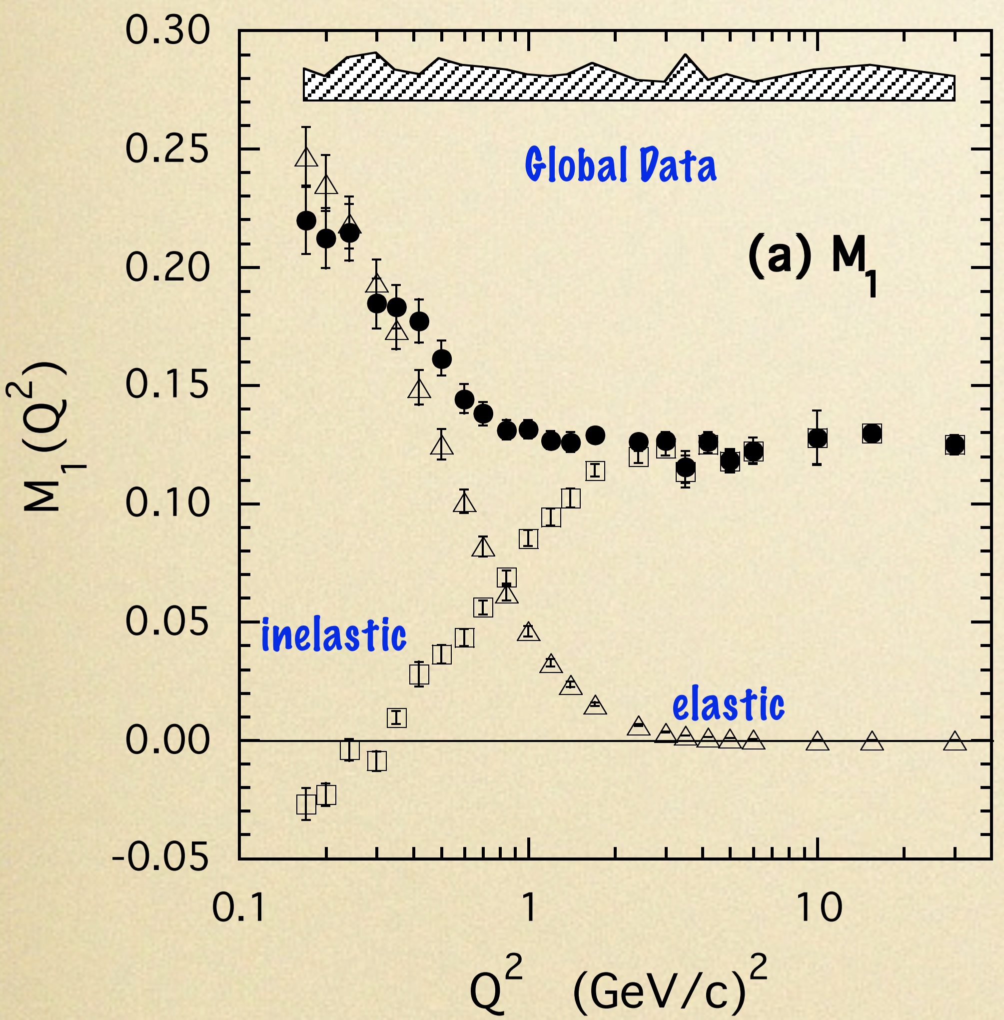
For EJ & Bj Sum rules

Larin & Vermaseren (1991)
Larin, Ritbergen & Vermaseren (1997)
Baikov, Chetyrkin & Kuhn (2010)

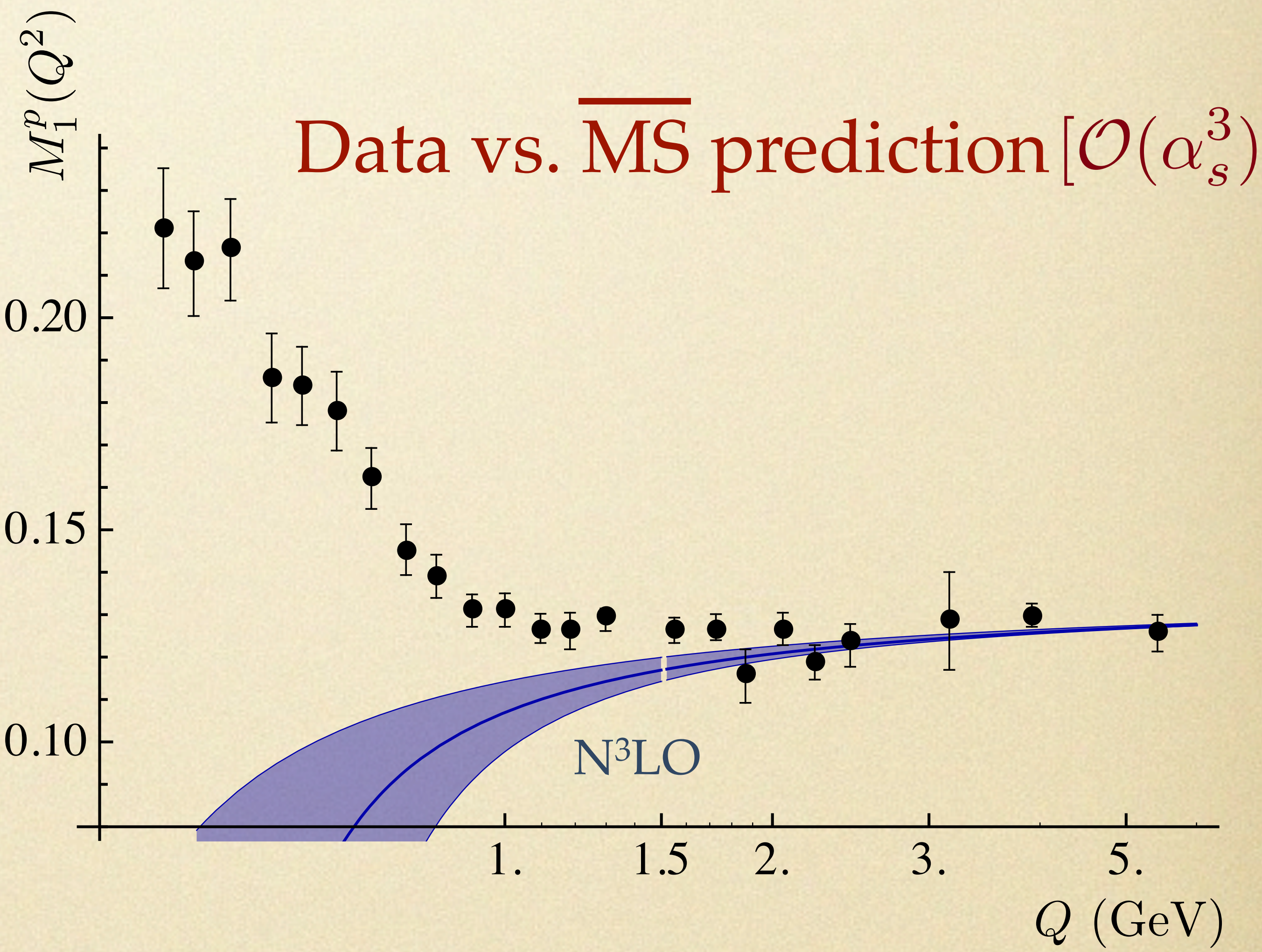
- Wilson Coefficients at twist-2 are known to four loops
- g_A is the neutron beta-decay constant $\quad g_A = 1.2695 \pm 0.0029$
- a_8 is the hyperon decay constant $\quad a_8 = 0.572 \pm 0.019$ — Campanario, Pineda (2005)
- other matrix elements need to be fitted from the data
- data is available for wide range of Q^2 values

Data

Ellis-Jaffe Sum Rule



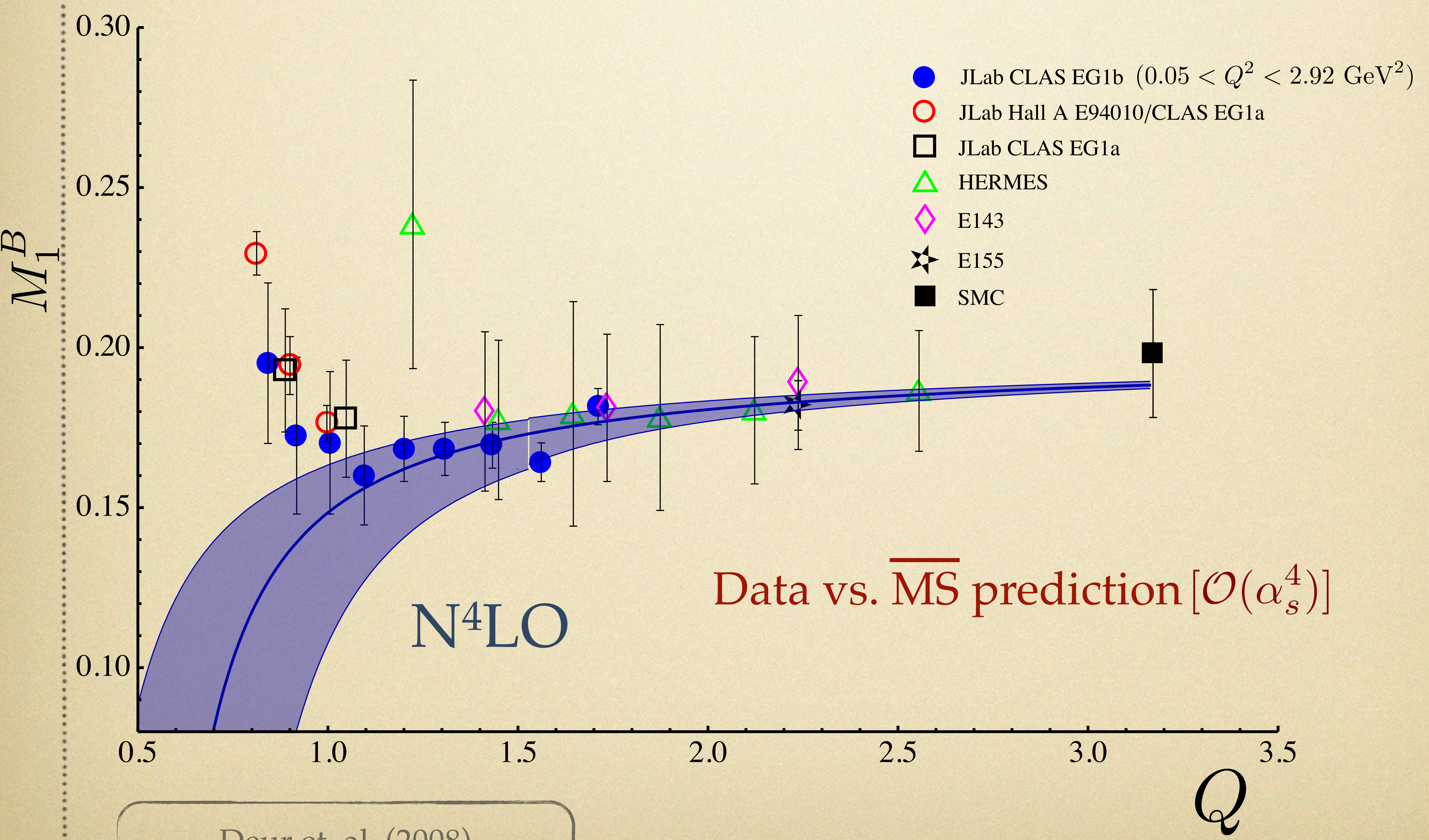
Osipenko et. al. (2005)



Fixed $a_0 = 0.140$ to agree with data at large Q

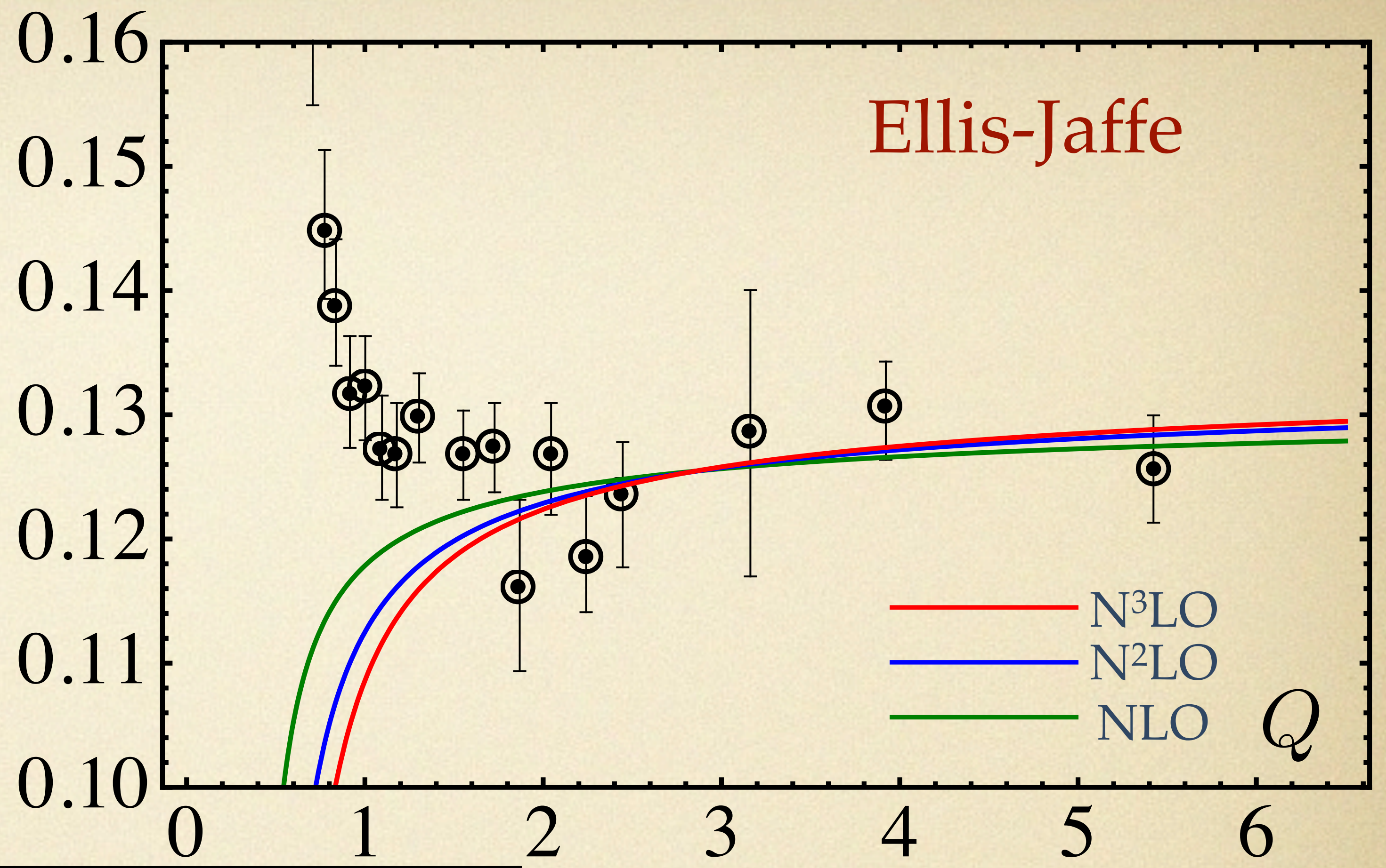
Data

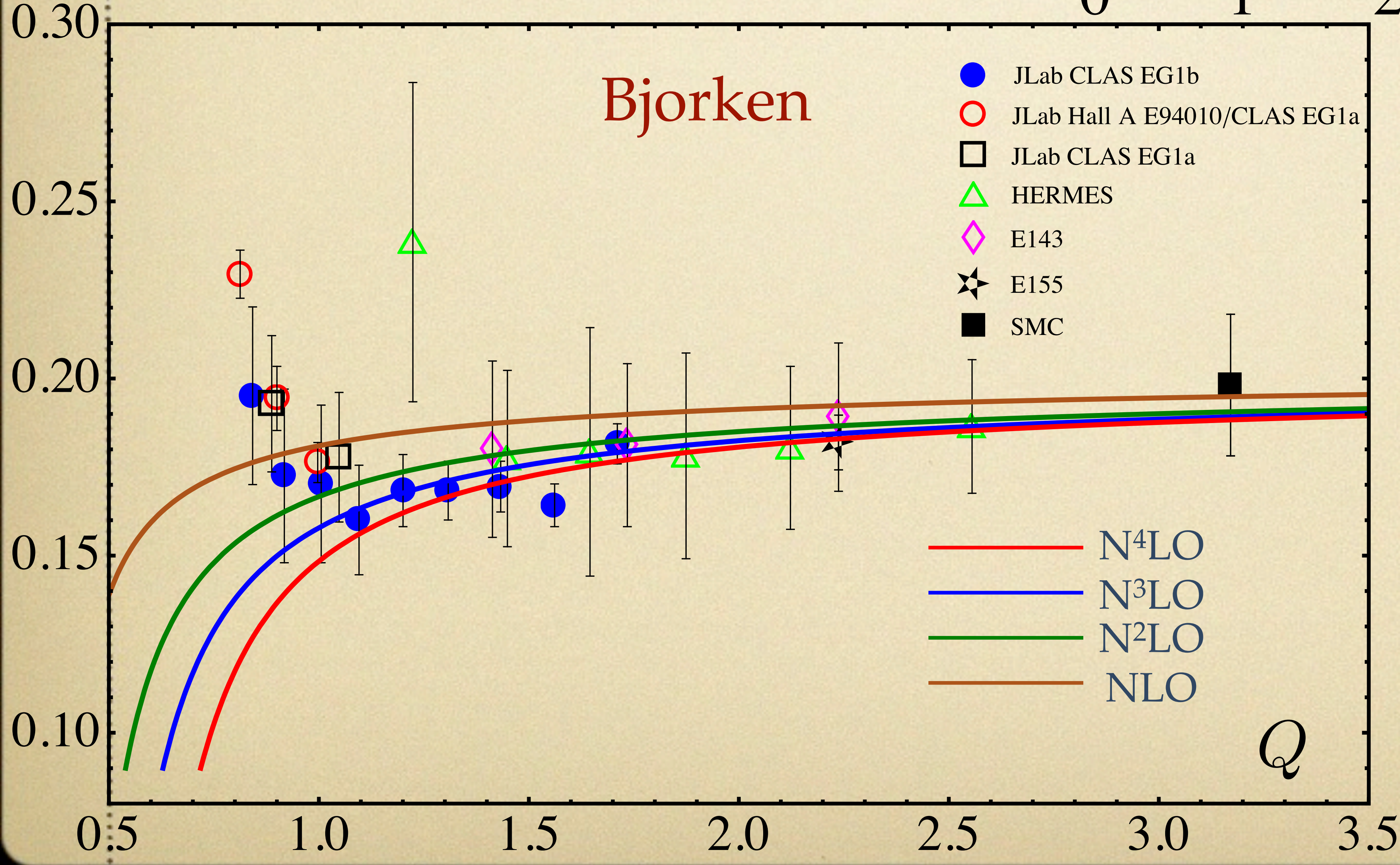
Bjorken Sum Rule



Deur et. al. (2008)

Order by order Comparison with $\overline{\mathrm{MS}}$





perturbation series diverges due to renormalon ambiguity or factorial growth

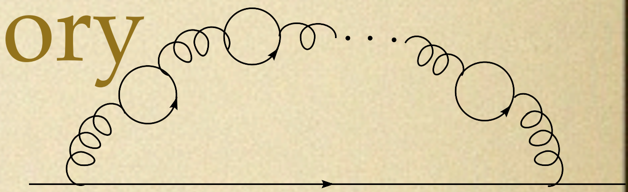
Thus power corrections are also unstable in $\overline{\mathrm{MS}}$ OPE !!

Renormalons

in a snapshot

for a detailed review, See Beneke (1999)

- Factorial growth in pert. series $C = \sum_n a_n \alpha_s^{n+1}$ $a_n \sim n!$
- Arise due to IR-sensitivities in the loop-integral: in $\overline{\text{MS}}$ IR-region is included in the loop integrals
- can be calculated to all orders in pert. theory with large n_f approximation



- strength of the renormalon ambiguity can be quantified perturbatively

Hoang, AJ, Stewart, Scimemi (2008)

$$C = P_p \, \Lambda^p_{\text{QCD}} \times \text{div.integral}$$

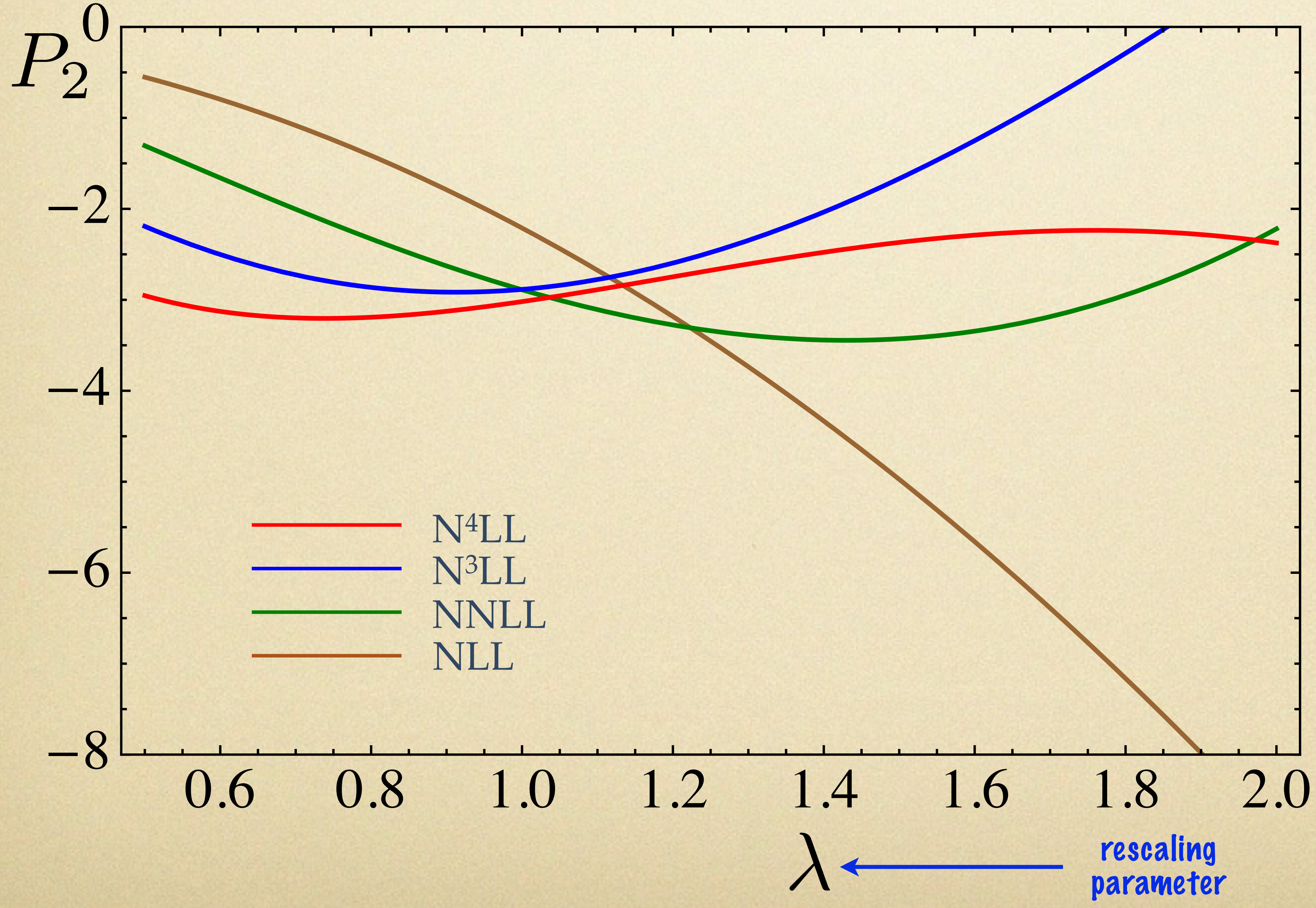
perturbative

Test for renormalon

via renormalon sum rule

Hoang, AJ, Scimemi, Stewart (to appear)

Strength of renormalon in C_B



same is true for C_0

Finiteness of Observable

renormalon cancellations in OPE

$$M_1^{p/n}(Q) = \left[\pm \hat{C}_B(Q)\left(\frac{1}{12}g_A + \frac{1}{36}a_8\right) + \hat{C}_0(Q)\frac{\hat{a}_0}{9}\right]$$
$$-\frac{1}{Q^2}\left[\left(\pm\frac{2}{27}\hat{f}_3 + \frac{2}{81}\hat{f}_8\right)[\alpha_s(Q)]^{\gamma_0^{ns}/(2\beta_0)} + \frac{8}{81}\hat{f}_0[\alpha_s(Q)]^{\gamma_0^{s}/(2\beta_0)}\right]$$
$$+\frac{1}{Q^4}\left[\pm h_B + h_0\right]$$

renormalon cancellation

renormalon cancels between C_i and f_i

may have a renormalon canceling a subleading renormalon in $C_{B,0}$

$$M_1^B \equiv M_1^p(Q) - M_1^n(Q) = \hat{C}_B(Q)\frac{g_A}{6} - \frac{4[\alpha_s(Q)]^{\gamma_0^{ns}/(2\beta_0)}}{27Q^2}\hat{f}_3 + \frac{1}{Q^4}h_B$$

- renormalons in Wilson coefficients cancel against higher twist matrix elements

Luke, Manohar, Savage (1994)

Examples where this happens:

- heavy meson mass-splitting (B-B* and D-D*)
- R-ratio
- Ellis-Jaffe & Bjorken sum rule

MSR Scheme

Hoang, AJ, Scimemi, Stewart (2009)

a solution to the problem

- subtract the asymptotic growth from Wilson coefficients
- add it back to the corresponding matrix element at higher twist

$$C_i^{\mathrm{MSR}}(Q,R) = \hat{C}_i(Q) - \frac{R^2}{Q^2}\frac{\hat{C}_1(Q)}{\hat{C}_1(R)}\Big\{\hat{C}_0(R) - [\hat{C}_0(R)]_{LL}\Big\}$$

$$f_i^{\mathrm{MSR}}(R) = \hat{f}_i + \frac{R^2}{Q^2}\frac{a_i}{\hat{C}_1(R)}\Big\{\hat{C}_0(R) - [\hat{C}_0(R)]_{LL}\Big\}$$

R is the IR subtraction scale in the Wilson coefficient $(\Lambda_{\mathrm{QCD}} \lesssim R)$

MSR Scheme

scholium

- both Wilson coefficient and matrix element are now free of the renormalon ambiguity
- overall OPE does not change but gives reliable predictions, converges faster.
- power corrections will be of their true size ($\sim\Lambda_{QCD}$) and won't depend on the order in the perturbation theory
- new MSR OPE has features of Wilsonian OPE, i.e., Wilson coefficients contain power like terms
- matrix elements in MSR and $\overline{\text{MS}}$ are perturbatively related

R-evolution

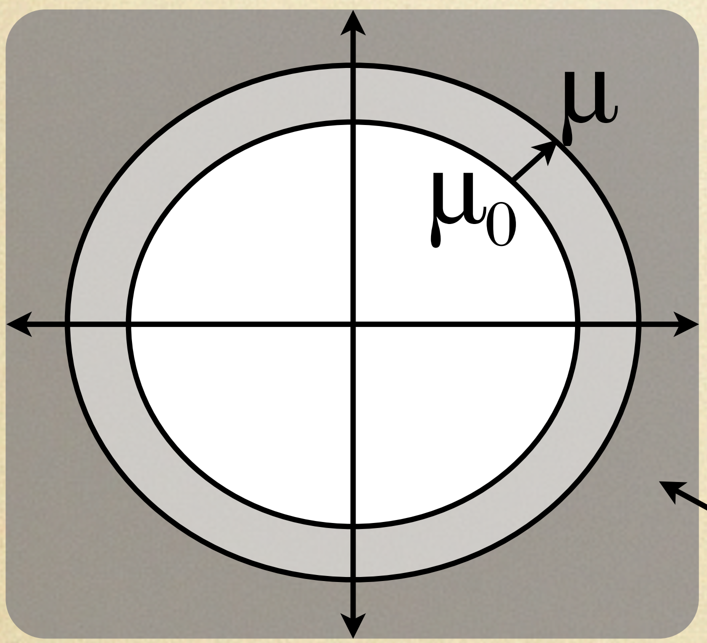
Hoang, AJ, Scimemi, Stewart (2008)

resumming IR-logs in Wilson coefficients

$$R\frac{d}{dR}C_0(Q,R) = -\frac{R^2}{Q^2}\hat{C}_1(Q)\gamma[\alpha_s(R)]$$

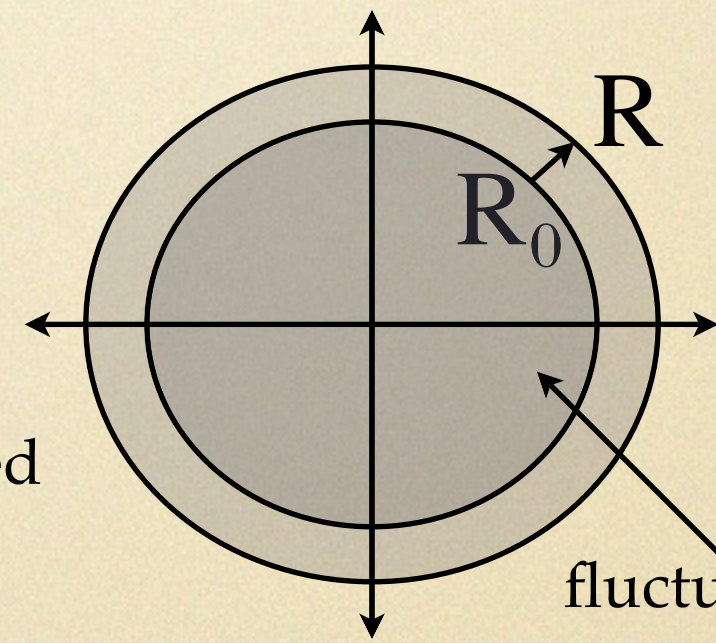
Solution of this equation resums large logs of R/Q

Analogy with μ-RGE



fluctuations absorbed in the Wilson Coefficients

increasing μ : less UV fluctuations in C and more in matrix elements



fluctuations absorbed in the matrix element

increasing R : more IR fluctuations in the matrix elements and less in C

OPE in MSR

$$
\begin{aligned}
M_1^{p/n}(Q) \quad &= \quad \Big[\pm C_B^{\mathrm{MSR}}(Q,R_0)\left(\frac{1}{12}g_A+\frac{1}{36}a_8\right)+C_0^{\mathrm{MSR}}(Q,R_0)\frac{\hat{a}_0}{9}\Big] \\
&\quad -\frac{1}{Q^2}\Bigg[\left(\pm\frac{2}{27}f_3^{\mathrm{MSR}}(R_0)+\frac{2}{81}f_8^{\mathrm{MSR}}(R_0)\right)[\alpha_s(Q)]^{\gamma_0^{ns}/(2\beta_0)} \\
&\quad +\frac{8}{81}f_0^{\mathrm{MSR}}(R_0)\,[\alpha_s(Q)]^{\gamma_0^{s}/(2\beta_0)}\Bigg]+\frac{1}{Q^4}\Big[\pm h_B+h_0\Big]
\end{aligned}
$$

$$
M_1^B \equiv M_1^p(Q)-M_1^n(Q) \quad = \quad C_B^{\mathrm{MSR}}(Q,R_0)\,\frac{g_A}{6}-\frac{4[\alpha_s(Q)]^{\gamma_0^{ns}/(2\beta_0)}}{27Q^2}f_3^{\mathrm{MSR}}(R_0)+\frac{1}{Q^4}h_B
$$

Lets compare theory predictions from leading twist in MSR scheme and then fit for matrix elements at higher twist

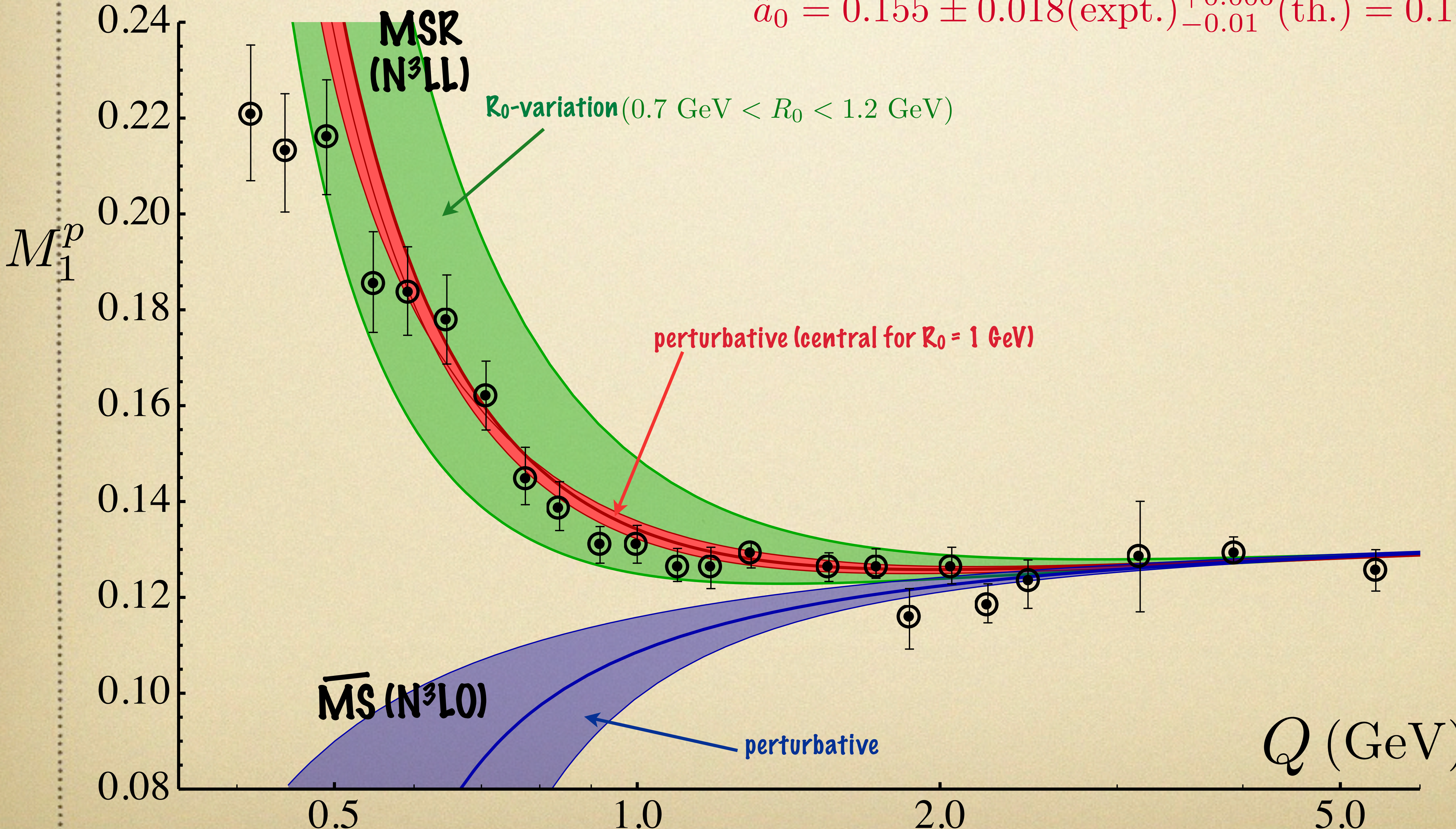
Comparison with Data

Ellis-Jaffe sum rule at leading twist in MSR

without a fit to power corrections (1 parameter)

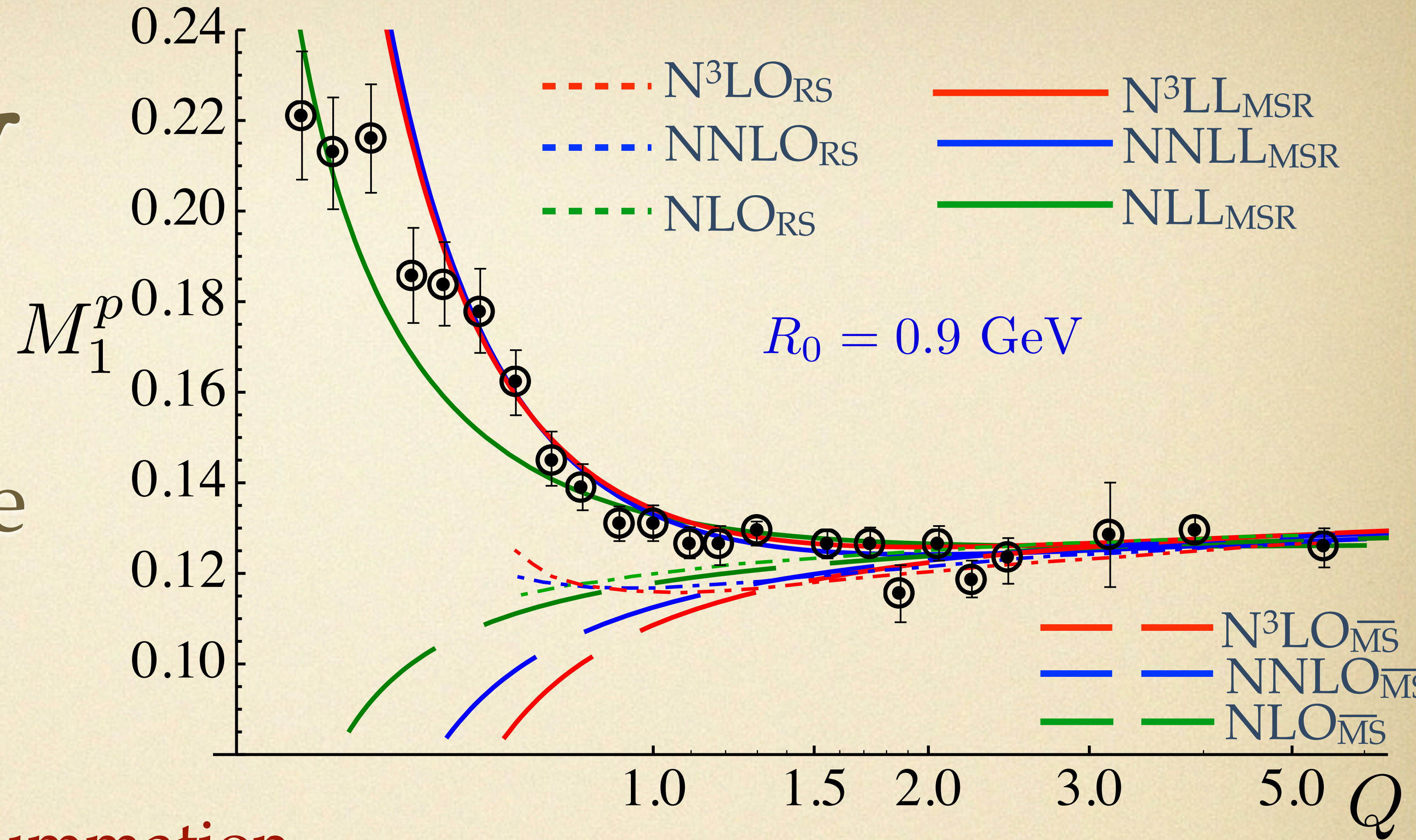
Naive fit with Q > 2GeV data only in $\overline{\text{MS}}$:

$\hat{a}_0 = 0.155 \pm 0.018(\text{expt.})^{+0.006}_{-0.01}(\text{th.}) = 0.155\ ^{+0.019}_{-0.021}$

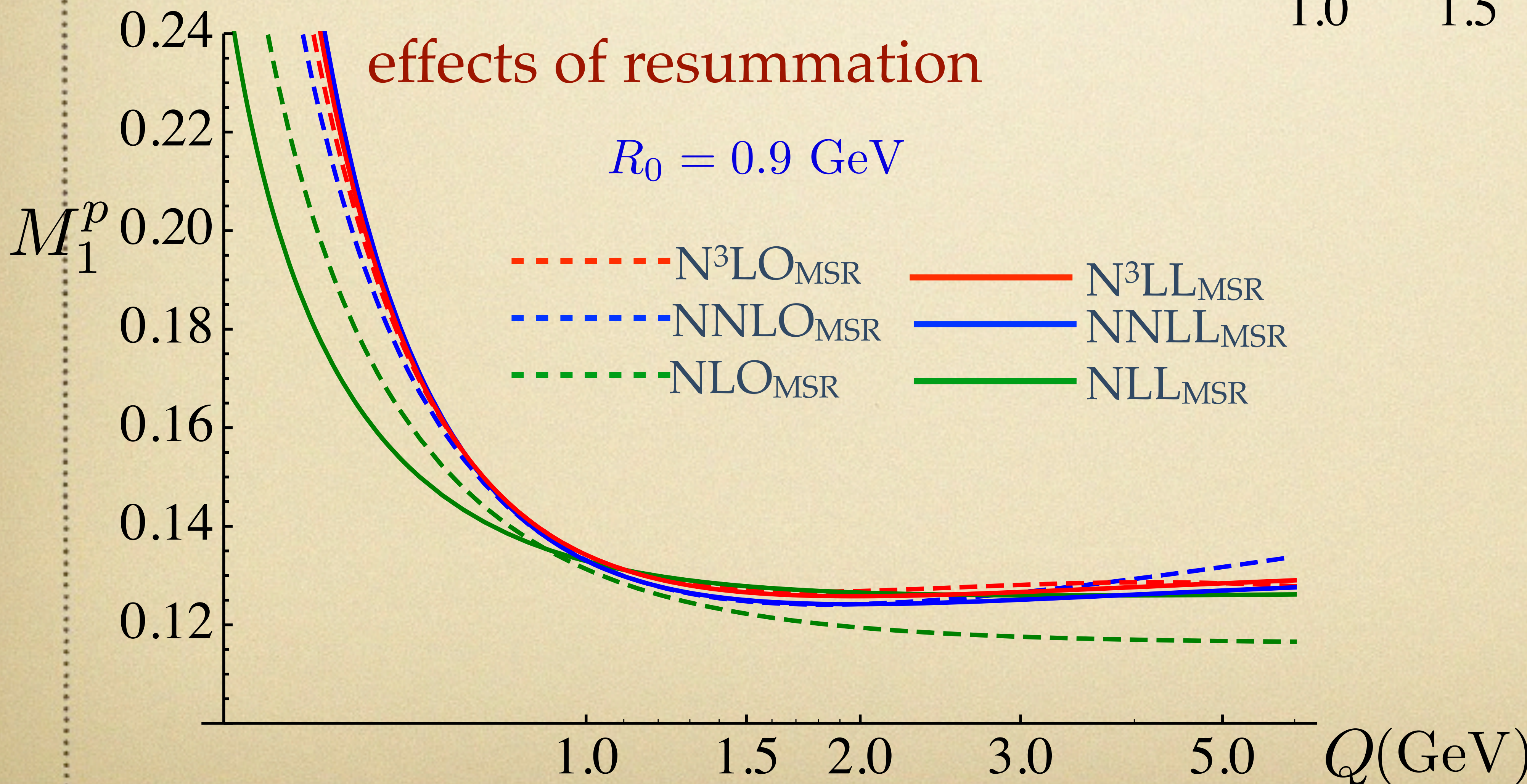


Order by Order

Ellis-Jaffe sum rule at leading twist



RS Scheme: Campanario, Pineda (2005)

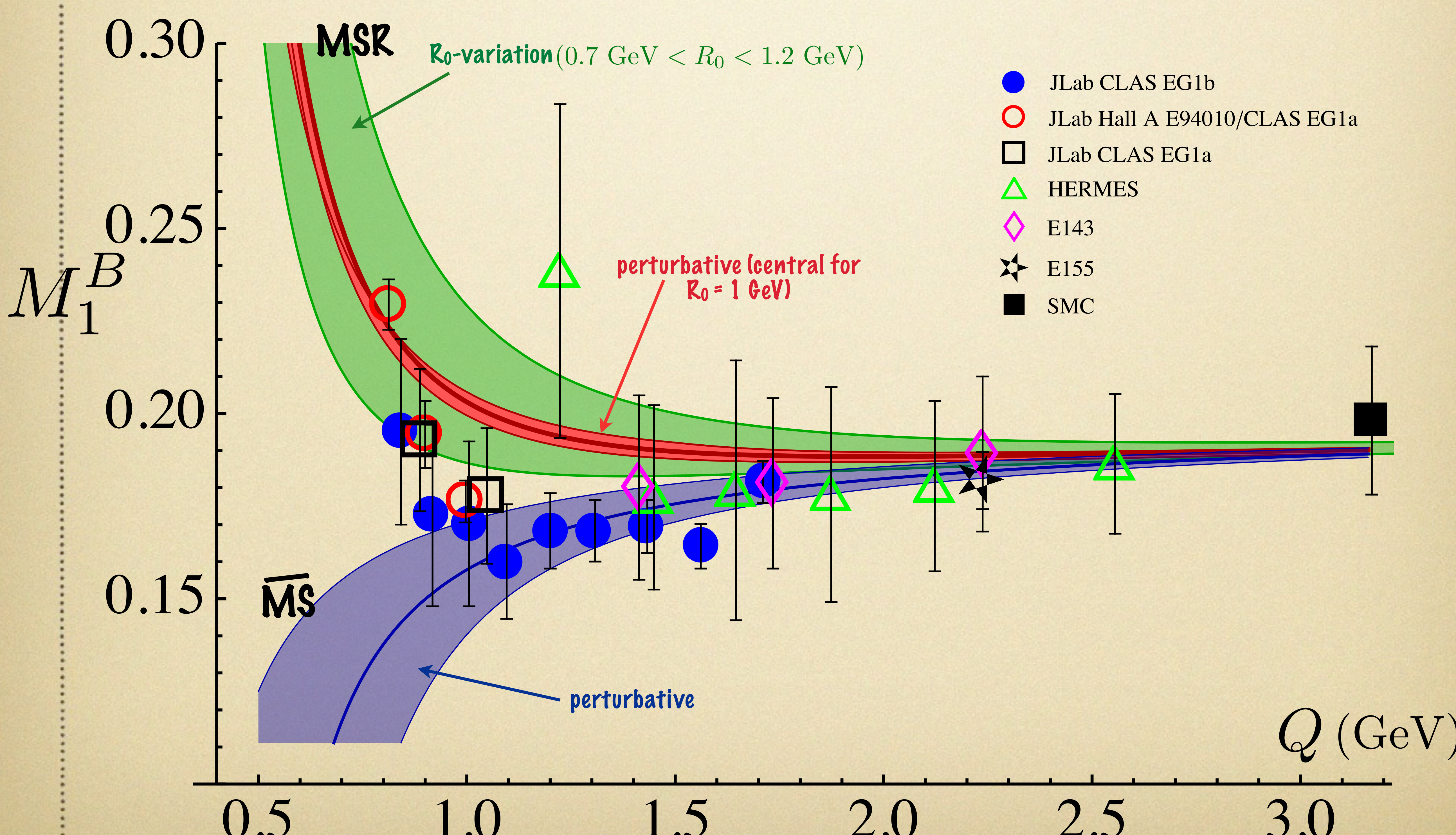


resummations matter when logs are large!

Comparison with Data

Bjorken sum rule at leading twist in MSR

without a fit to power corrections



Fits for power corrections

EJSR: MSR vs. MS

- we do 4-parameter fit to the global data for Ellis-Jaffe sum rule
- we take half the error as systematic and 100% correlation in systematic errors as our model for correlation matrix

$\overline{\text{MS}}$

order	$\frac{\hat{a}_0}{9}$	$\frac{-8}{9}\left(\frac{\hat{f}_3}{12}+\frac{\hat{f}_8}{36}\right)$ (GeV^2)	$\frac{-8}{81}\hat{f}_0$ (GeV^2)	h_B+h_0 (GeV^4)	χ^2/dof
tree	0.0022	0.0526	−0.0713	0.0107	1.18
$1-$ loop	0.0117	0.0593	−0.0603	0.0081	1.05
$2-$ loop	0.0138	0.0350	−0.0109	0.0039	1.05
$3-$ loop	0.0137	−0.0893	0.1899	−0.0132	1.54

MSR ($R_0 = 1$ GeV)

order	$\frac{\hat{a}_0}{9}$	$\frac{-8}{9}\left(\frac{f_3^{\text{MSR}}}{12}+\frac{f_8^{\text{MSR}}}{36}\right)$ (GeV^2)	$\frac{-8}{81}f_0^{\text{MSR}}$ (GeV^2)	h_B+h_0 (GeV^4)	χ^2/dof
tree	0.0022	0.0526	−0.0713	0.0107	1.18
NLL	0.0125	0.0399	−0.0713	0.0095	1.12
NNLL	0.0164	0.0568	−0.0912	0.0048	0.96
N^3LL	0.0156	0.0559	−0.0892	0.0050	0.97

Fits for power corrections

BjSR: MSR vs. $\overline{\text{MS}}$

- we do 2-parameter fit with only *JLab CLAS EG1b* data for Bjorken sum rule: latest data, largest data set from single experiment, and spans both pert. and non. pert. regions

- 100% correlation in correlated systematic errors is assumed

$\overline{\text{MS}}$

order	$\frac{-4}{27}\hat{f}$ (GeV^2)	h_B (GeV^4)	χ^2/dof
tree	-0.1751	0.0884	0.41
$1-$ loop	-0.0455	0.0318	0.38
$2-$ loop	-0.0045	0.0182	0.40
$3-$ loop	0.0116	0.0160	0.41

MSR ($R_0 = 1$ GeV)

order	$\frac{-4}{27}f^{\text{MSR}}$ (GeV^2)	h_B (GeV^4)	χ^2/dof
tree	-0.1751	0.0884	0.41
NLL	-0.0857	0.0302	0.38
NNLL	-0.0281	-0.0083	0.45
N^{3}LL	-0.0355	-0.0034	0.43

large errors and large correlations !!

Fits for power corrections

Error matrix from expt. & theory errors

$$
\begin{array}{cccccc}
\frac{\hat{a}_0}{9} & \frac{-8}{9}\left(\frac{f_3^{\mathrm{MSR}}}{12}+\frac{f_8^{\mathrm{MSR}}}{36}\right) & \frac{-8}{81}f_0^{\mathrm{MSR}} & h_B+h_0 & & \\
\end{array}
$$

$$
\left[\begin{array}{cc}
\begin{pmatrix}
6.33\times10^{-6} & 1.66\times10^{-7} & -1.21\times10^{-5} & 3.63\times10^{-6}\\
1.66\times10^{-7} & 7.13\times10^{-5} & -1.1\times10^{-4} & 1.31\times10^{-5}\\
-1.21\times10^{-5} & -1.1\times10^{-4} & 2.38\times10^{-4} & -3.91\times10^{-5}\\
3.63\times10^{-6} & 1.31\times10^{-5} & -3.91\times10^{-5} & 7.73\times10^{-6}
\end{pmatrix} & 0 \\
0 & \begin{pmatrix}
5.45\times10^{-3} & -3.04\times10^{-3}\\
-3.04\times10^{-3} & 1.79\times10^{-3}
\end{pmatrix}
\end{array}\right]
$$

$$
\begin{array}{cc}
\frac{-4}{27}f_3^{\mathrm{MSR}} & h_B
\end{array}
$$

$$\frac{\hat{a}_0}{9} = 0.0156 \pm 0.0025(\mathrm{expt.})^{+0.0006}_{+0.0005}(\mathrm{th.})$$

$$\frac{-4}{27}f_3^{\mathrm{MSR}} = -0.036 \pm 0.07(\mathrm{expt.})^{+0.01}_{-0.01}(\mathrm{th.})$$

$$\frac{-8}{9}\left(\frac{1}{12}f_3^{\mathrm{MSR}} + \frac{1}{36}f_8^{\mathrm{MSR}}\right) = 0.056 \pm 0.008(\mathrm{expt.})^{+0.009}_{-0.005}(\mathrm{th.})$$

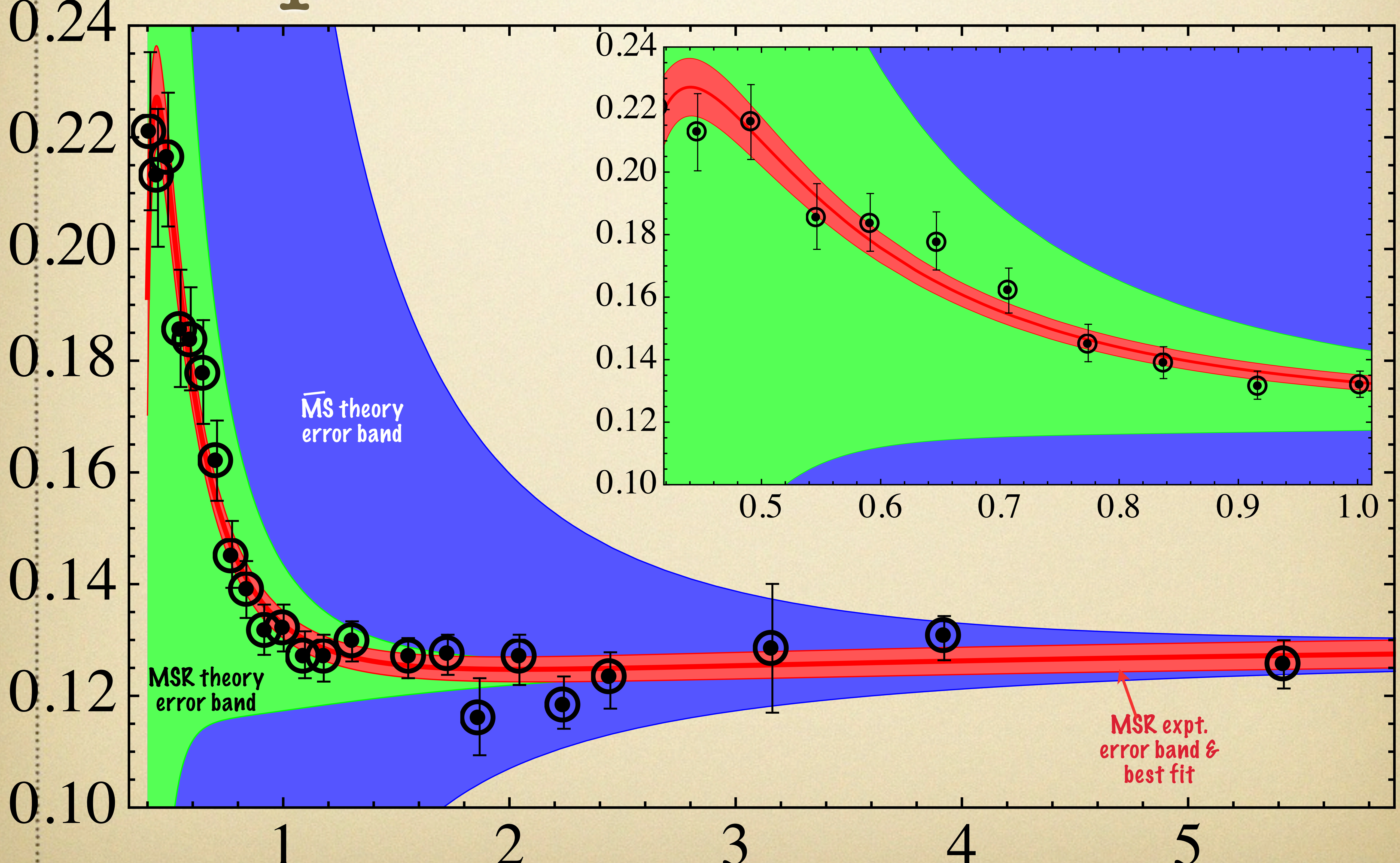
$$h_B + h_0 = 0.005 \pm 0.003(\mathrm{expt.})^{+0.006}_{-0.001}(\mathrm{th.})$$

$$\frac{-8}{81}f_0^{\mathrm{MSR}} = -0.089 \pm 0.015(\mathrm{expt.})^{+0.011}_{-0.027}(\mathrm{th.})$$

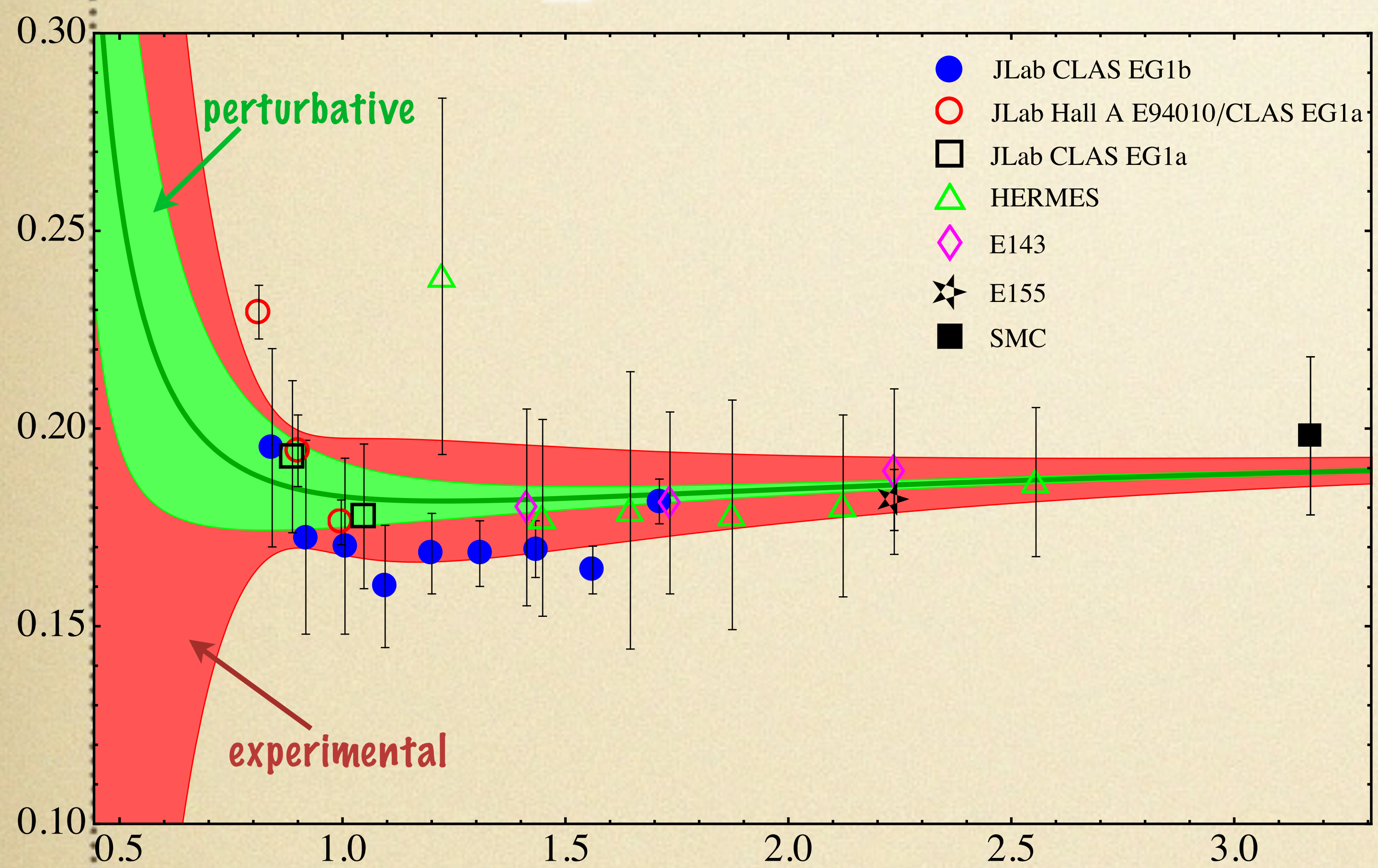
$$h_{\mathrm{B}} = -0.003 \pm 0.04(\mathrm{expt.})^{+0.005}_{-0.003}(\mathrm{th.})$$

all f_i are shown for $R_0 = 1$ GeV

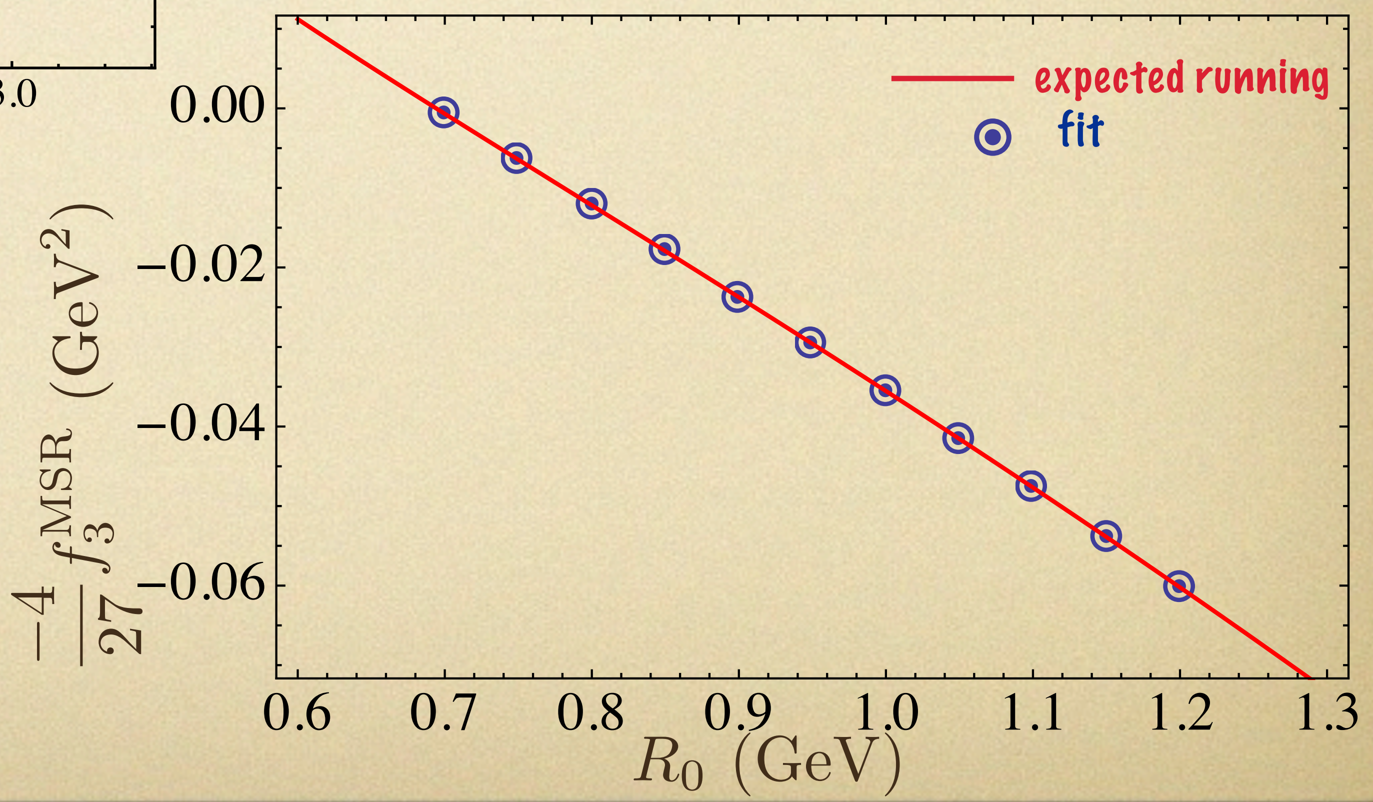
EJSR in MSR including power corrections



BjSR in MSR including power corrections



here, theory errors are smaller than experimental



Fitted power corrections run as expected.

Same is true in EJSR

Conclusions

- Obtained stable OPE in new MSR scheme
- Resummation of logs of Q/R using the R-RGE
- Obtained reliable fits for higher twist matrix elements